Quadratics (1)

1. **(a)** State the relation between a, b and c such that the equation $ax^2 + bx + c = 0 \ (a \neq 0)$ has equal roots.

(b) If the equation $a^2x^2 + 3abx + ac + 2b^2 = 0 \quad (a \neq 0)$ has equal roots,
show that the roots for the equation $ac(x + 1)^2 = b^2x \ (a, b, c \neq 0)$ are equal.

2. If x is real, show that the expression $y = \frac{x^2+x+1}{x+1}$ does not have a value between -3 and 1 .

3. Let the equations $x^2 + ax + b = 0$ and $x^2 + cx + d = 0 \ (b \neq d)$ have one non-zero common root.
Form an equation with the other roots of these equations.

4. If a, b and c are real numbers, show that the roots of the equation $(a - b - c)x^2 + ax + b + c = 0$ is real.
If one of the roots is twice the other, show that $b + c = \frac{a}{3}$ or $\frac{2a}{3}$.

5. If α_1 and β_1 are the roots of the equation $x^2 + 2ax + b^2 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2cx + d^2 = 0$, show that :

(a) If $\alpha_1 + \alpha_2 = \beta_1 + \beta_2$, then $a^2 + d^2 = b^2 + c^2$,

(b) If $\alpha_1\alpha_2 + \beta_1\beta_2 = 0$, then $b^2d^2 = a^2d^2 + b^2c^2$.

6. If the equation $ax^2 + bx + c = 0 \ (a \neq 0)$ has real roots, show that the equation

$$(a + c - b)x^2 - 2(a - c)x + (a + c + b) = 0$$

has also real roots.

Show that if α and β are the roots for the first equation, then the product of roots of the second equation is

$$\frac{(1 - \alpha)(1 - \beta)}{(1 + \alpha)(1 + \beta)}$$

7. If α, β are roots of the equation $x^2 + px + q = 0$ and α_1, β_1 are roots of the equation $x^2 + p_1x + q_1 = 0$.
Express $(\alpha - \alpha_1)(\alpha - \beta_1) + (\beta - \alpha_1)(\beta - \beta_1)$ in terms of p, q, p_1 and q_1.

8. Show that the expression $\frac{5}{2x^2+3x+3}$ is positive and find its greatest value.
Hence find the smallest values of $\frac{6x^2+9x+4}{2x^2+3x+3}$.
Sketch the functions of $2x^2 + 3x + 3$, $\frac{5}{2x^2+3x+3}$, $\frac{6x^2+9x+4}{2x^2+3x+3}$ together on the same graph.